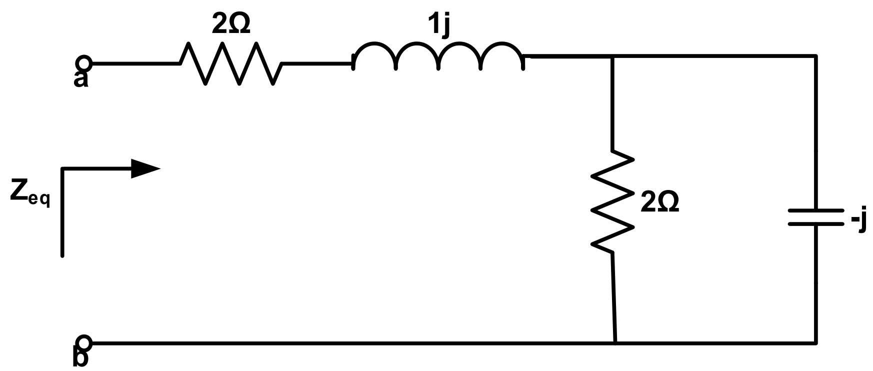
Electrical and Computer Engineering Department

Network Analysis I, ENEE2301

Name: Second Exam 2017-05-14

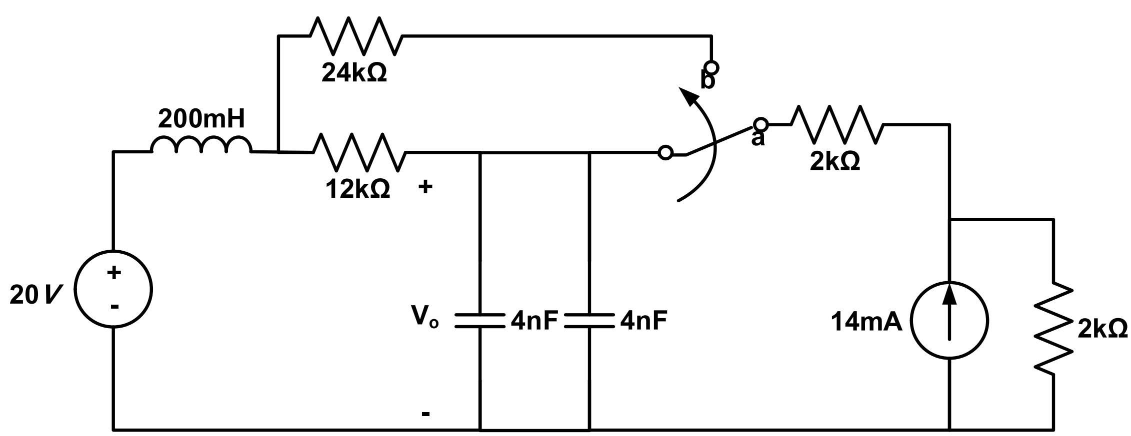
No.: Sec:

Q1) Find $\mathbf{Z_{eq}}$ between (**a** and **b**) in phasor form $|Z_{eq}|\angle\theta$? (10 marks)



$$Z_{eq} = \frac{-j*2}{2-j} + 2 + j = 2.4 + 0.2j = 2.4\angle 4.37\Omega$$

Q2)



For all the time prior to zero, the switch is at position (a), at t=0 the switched was moved to position (b).

- The circuit can be simplified as

Convert the current source 14mA to 28V voltage source

a) Find $V_o(0^+)$? (5 marks)

$$V_o(0^+) = V_c(0^-) = \frac{V_c(0^-) - 20}{12k} + \frac{V_c(0^-) - 28}{4k} = 0$$

$$V_c(0^-) = \frac{20 + 3 * 28}{4} = 26V$$

$$i_L(0^+) = \frac{20 - 28}{16k} = -0.5mA$$

$$i_C(0^+) = i_L(0^+) = C\, dv/dt = -0.5mA$$

$$V_L(0^+) = Ldi/dt = V_o(0^+) + (12//24)k * i_L(0^+) - 20 = 2V$$

$$V_o(\infty) = 20V$$

$$i_C(\infty) = 0A$$

b) Show that type of damping is under damped? (10 marks)

$$\alpha = \frac{R}{2L} = \frac{12k//24k}{2(0.2)} = 20000$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2 * 8 * 10^{-9}}} = 25000$$

$$\alpha^2 < \omega^2$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha_o^2} = \sqrt{625 * 10^6 - 400 * 10^6} = 15000$$

c) Find dv_o/dt (5 marks)

$$dv_o/dt = \frac{1}{c} i_c = \frac{1}{c} i_L$$

$$= \frac{1}{c}(i_L(\infty) + (B_1 cos\omega_d t + B_2 sin\omega_d t)e^{-\alpha t})$$

$$= \frac{1}{8 * 10^{-9}}(B_1 cos15000t + B_2 sin15000t)e^{-2000t}$$

$$\frac{1}{c} i_c(0) = \frac{1}{8 * 10^{-9}}(i_L(\infty) + B_1)$$

$$-0.5 * 10^{-3} = B_1$$

$$di_l/dt\,(0) = \frac{1}{0.2}(-20000 * -0.5 * 10^{-3} + 15000B_2) = \frac{V_L(0^+)}{L} = \frac{2}{0.2}$$

$$B_2 = 53.3 * 10^{-3}$$

$$dv_o/dt = \frac{1}{8 * 10^{-6}}(-0.5cos15000t - 53.33sin15000t)e^{-2000t}$$

d) For $t \geq 0$ find $V_o(t)$? (10 marks)

$$V_o(t) = V_c(\infty) + (B_1 cos15000t + B_2 sin15000t)e^{-2000}$$

$$V_o(0) = V_c(\infty) + B_1$$

$$26 = 20 + B_1$$

$$B_1 = 6$$

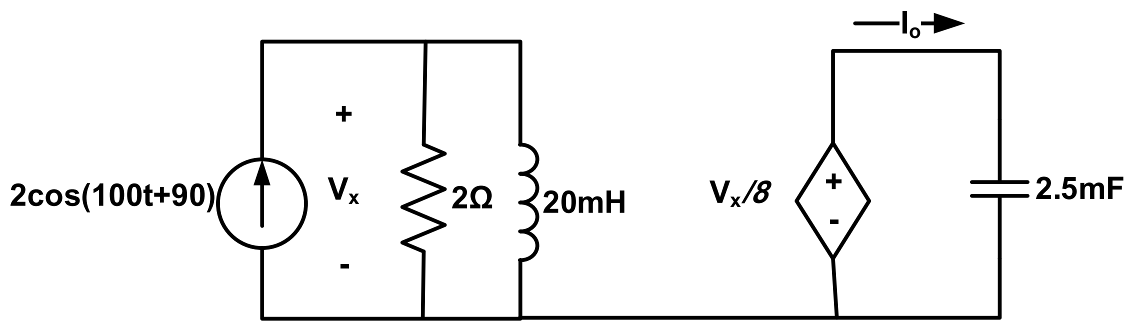
$$dv_o/dt\,(0) = -20000B_1 + 15000B_2$$

$$\frac{-0.5 * 10^{-3}}{8 * 10^{-9}} = -20000(6) + 15000B_2$$

$$B_2 = 3.833$$

$$V_o(t) = 20 + (6cos15000t + 3.833sin15000t)e^{-2000}$$

Q3) Find the phasor current **Io**? (20 marks)



$$V_x = \frac{I_{in}}{Z_{RL}} = 2\angle 90 * \frac{4j}{2+2j} = 2\angle 90 * \sqrt{2}\angle -45 = 2\sqrt{2}\angle 45V$$

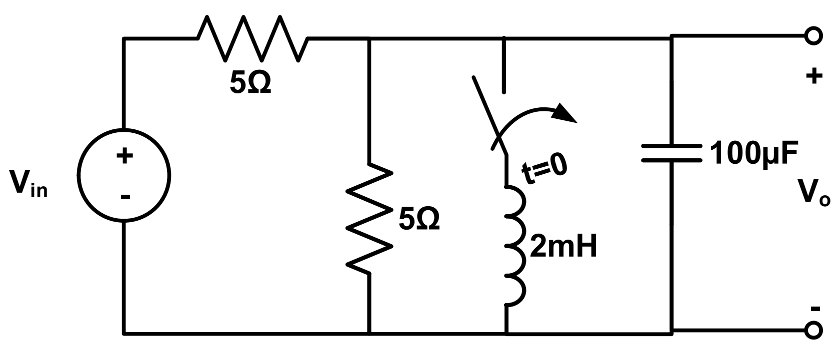
$$I_o = \frac{V_x}{8} / \frac{1}{0.25j} = \frac{\sqrt{2}}{16}\angle -45$$

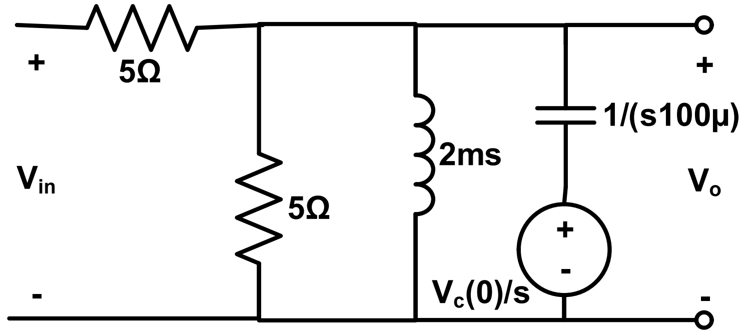
Not required

$$I_o = \frac{\sqrt{2}}{16}\cos(100t - 45)\,A$$

Q4) The switch has been open for a long time and it is closed at t=0.

a) Find the transfer function H(s)= $V_o(s)/V_{in}(s)$ after closing? (10 marks)
b) If V_{in}=10V, find $V_o(t)$ using the transfer function H(s) derived in the previous part? (10 marks)





$$V_c(0^-) = \frac{5}{5+5}V_{in} = 0.5V_{in}$$

$$\frac{V_o}{R} + \frac{V_o - 0.5V_{in}/s}{1/sc} + \frac{V_o}{sL} + \frac{V_o - V_{in}/s}{R} = 0$$

$$sLV_o + V_o s^2 RLC - 0.5sRLCV_{in} + RV_o + V_o sL - LV_{in} = 0$$

$$H(s) = \frac{V_o}{V_{in}} = \frac{(0.5sRLC + L)}{(s^2RLC + 2sL + R)} = \frac{(0.5s + \frac{1}{RC})}{\left(s^2 + \frac{2}{RC}s + \frac{1}{LC}\right)}$$

$$H(s) = \frac{(0.5s + 2000)}{(s^2 + 4000s + 5 * 10^6)}$$

$$= \frac{0.5(s + 2000) + 1000}{(s + 2000)^2 + 1000^2}$$

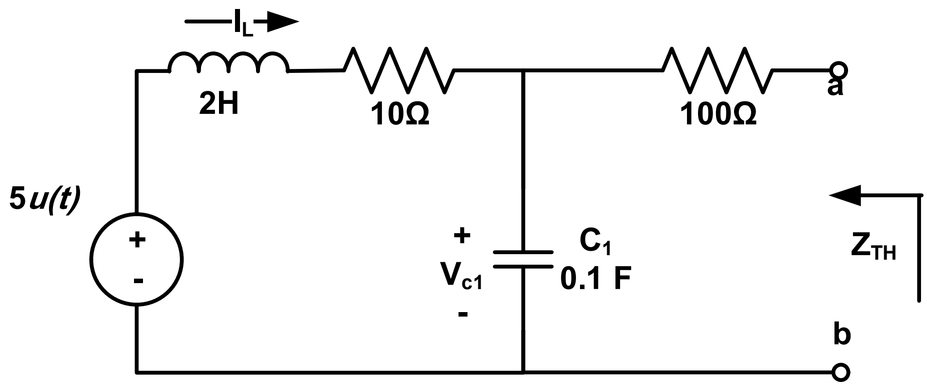
$$\frac{0.5(s+2000)}{(s + 2000)^2 + 1000^2} + \frac{1000}{(s + 2000)^2 + 1000^2}$$

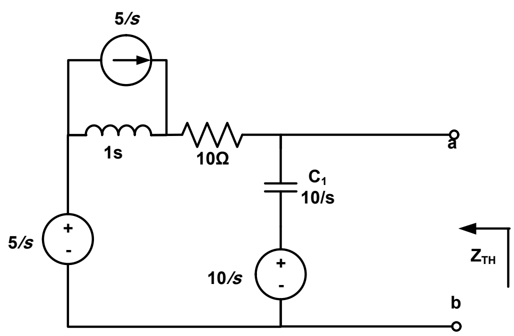
$$\frac{V_o}{V_{in}}(t) = 0.5\big(cos(1000t) + 2sin(1000t)\big)e^{-2000t}u(t)$$

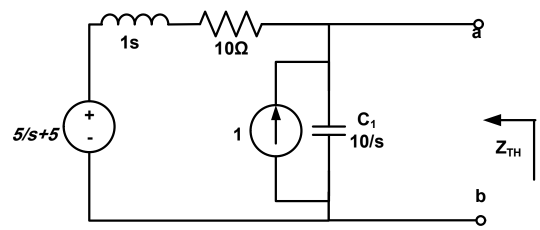
$$V_o(t) = 5\big(cos(1000t) + 2sin(1000t)\big)e^{-2000t}u(t)$$

Q5) If $V_{c1}(0^-)$=10V and $I_L(0^-)$=5A

a) Redraw the circuit in s-domain. (8 marks)
b) Find the Norton equivalent circuit between (a and b) in the s-domain? (12 marks)

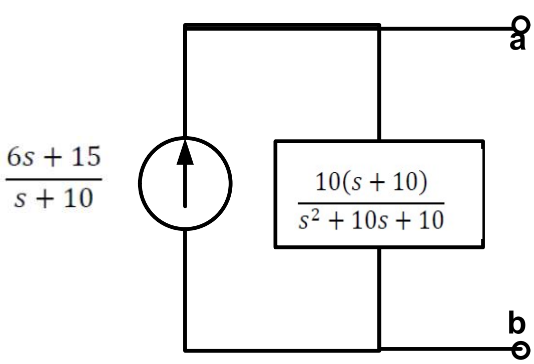






$$Z_{th} = \frac{\frac{10}{s}(s+10)}{\frac{10}{s}+(s+10)} = \frac{10(s+10)}{s^2+10s+10}$$

$$I_n = \frac{5s+5}{s+10} + 1 = \frac{5s+5+s+10}{s+10} = \frac{6s+15}{s+10}$$



Not required

$$I_n = 6\delta(t) - 45e^{-10t}$$

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